A Skeptic's View of Unstirred Components

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***Abstract*—In the practice and research on reverberation chambers, the concept of unstirred field components have often been invoked as soon as empirical distributions fail to comply with goodness-of-fit tests applied to data collected at a frequency of operation far beyond the lowest usable frequency. The current explanation for this phenomenon is that under certain conditions, the field samples generated by means of a stirring procedure are not characterized by a zero average-value, but actually present a deterministic offset term that is linked to a line-of-sight contribution. In this paper, we prove that this practice is not sound, as it does not acknowledge the fact that even a very low but non-zero residual correlation between the samples is enough to put in jeopardy the validity of the limits imposed by most hypothesis-test statistics, and hence their ability in properly detecting any constant contribution. An alternative approach is here proposed, based on the analysis of the variability of the line-of-sight contribution estimate, capable of accounting for the residual correlation in a reverberation chamber. Experimental results are presented to support the validity of our approach, exposing the critical use of goodness-of-fit tests as currently applied.**

***Index Terms*—Reverberation chambers, Statistics, Hypothesis test, Unstirred components.**

I. Introduction

Reverberation chambers (RCs), when operated at a sufficiently high frequency jointly with a stirring technique, are expected to be characterized by a field spatial distribution following a Gaussian probability law at any point within a volume of space, with equal moments, i.e., stationary in space [1], [2]. In fact, the practice of RCs is not so close to this ideal depiction, as non-agreements appear at different levels: not only field samples appear not to agree flawlessly with a Gaussian probability distribution, but even when they do, they may happen to present a non-zero average value. This phenomenon has been studied in several atypical (with respect to the usual practice for EMC tests) configurations [3], [4] where a direct line-of-sight (LoS) contribution between the excitation antenna and the field probe is established on purpose, leading to a fixed contribution negligibly affected by the stirring technique.

Clearly, admitting the existence of this LoS term implies that the field statistics will be skewed with respect to the ideal diffuse case. Perhaps because of this state of affairs, in several works presented in the last few years when the conditions expected to lead to an ideal diffuse field are reasonably true, if field samples do not comply with their expected behavior the presence of a LoS term has been invoked [5], [6]. In other words, it is considered that if the frequency of operation of the chamber is well above the lowest usable frequency (LUF) [7] and that the stirrer respects certain empirical constraints [8], the only way to explain the existence of non-ideal field statistics is to invoke the presence of LoS contributions. In practice, this is regarded as due to a non-optimal positioning and orientation of the excitation antenna, allowing the establishment of a direct propagation path between this antenna and the probe one. As this propagation model implies that the LoS contribution is weakly affected by the stirrer (as at least one path do not cross it), the LoS term is referred to as an unstirred component.

We are actually very skeptical of this explanation: it is hard to imagine non-negligible LoS terms appearing when the source antenna is not pointed directly towards the probe one. As it will be shown in the remainder of this paper, there are sound arguments that point to the fact that the methods currently used to detect the presence of unstirred components are not statistically nor physically justified. Our analysis proceeds from an original description of the behaviour of an RC as a multivariate random generator: this leads to a sounder definition of the correlation between samples. Experimental results interpreted through this model show the presence of residual weak correlation levels even in conditions deemed to be ideal. This fact is shown to be sufficient to lead to wrong conclusions on the presence of LoS contributions.

Our statistical description allows to define in a very natural manner an alternative hypothesis test that is expected to lead to a better decision for the existence of unstirred components. Experimental results are shown to comply in a very effective way with the theoretical behavior predicted by our model, thus suggesting that it can be regarded as closer to the real behavior of a real-life RC.

II. Reverberation chambers as multivariate random generators

Reverberation chambers are often regarded as random field generators [2]. When field probes are used to assess the statistical behavior of any scalar components of the electric field at a given position, the sequence of values generated by means of the chosen stirring technique is usually regarded as a random sequence. In particular, the field is modeled as a univariate random variable $x \in \mathbb{C}$, while the N_s samples obtained by operating the stirring technique are regarded as random realizations of this random variable. In the context of the present paper, stirring techniques will be assumed to be operated through discrete steps, for the sake of simplicity. The continuous case can be eventually inferred from our derivation.

The random sequence $\{x_i\}$ (corresponding, e.g., to a scalar field component) is typically analyzed by estimating its first two empirical moments

$$\hat{\mu}_x = \langle x_i \rangle_{N_s} \quad (1a)$$

$$\hat{\sigma}_x^2 = \frac{N_s}{N_s - 1} \left\langle |x_i - \hat{\mu}_x|^2 \right\rangle_{N_s} \quad (1b)$$

with $\langle y_i \rangle_N$ the arithmetic average of the N-sample sequence $\{y_i\}$. The random sequence is further studied by considering its empirical probability distribution function and its autocorrelation function (ACF), defined as

$$\phi(k) = \left(\sum_{i=1}^{N_s} |x_i|^2 \right)^{-1} \sum_{i=1}^{N_s} x_i x_{i+k}^{\star} \quad , \quad (2)$$

having chosen its normalized definition. The first sum is to be taken circularly over the entire set of samples, as these usually form a periodic sequence, depending on the stirring technique.

Random samples thus generated are expected to comply with a series of constraints dictated by the asymptotic model presented in [9]. If any scalar component of the electric field is taken at turn to be the $\{x_i\}$, then a sound approach is to apply goodness-of-fit (GoF) tests, intended to check the consistency of the samples with a null-hypothesis that requires them to follow a zero-average normal law. The samples are subsequently regarded as iid random variables, i.e., their ACF is considered to well approximate a Kronecker delta, i.e., to fall to negligible values for $k \neq 0$.

This approach has several drawbacks and limitations: first of all, it is applied in a local way, i.e., the results of the test can differ from two different positions within the same chamber. Such an outcome is critical, as it is actually inconsistent with the spatial stationarity of the random properties of the field distribution within an ideal RC [9], [1]. Secondly, the assumption that the field be stationary is physically justified only when moving in space, and not when a stirring technique is applied and the field observed at the same position. Clearly, the iid assumption is the ideal case of reference, but their is no physical reason to expect the field samples generated by, e.g., a mechanical stirrer to be iid. The same remark holds for the modeling of the random sequence as a time series, as suggested in [10]: regarding the correlation between two consecutive samples to be independent of the absolute position of the stirrer is clearly not physically justified.

We should add to these remarks a further fact. As the field is probed at different positions, the hypothesis test can yield different and inevitably contrasting conclusions. Should this be regarded as a token of the fact that the random properties of the field are not stationary in space, or rather that the application of GoF methods to a local level is not the right procedure? In fact, the former conclusion is not consistent with the original constraints imposed by the ideal behavior of an RC: as a result, it would be necessary to regard a chamber as non-ideal as soon as GoF tests were negative over some locations. As a matter of fact, why should one consider that a sequence with non-zero average value is regarded as unacceptable, while accepting that the field moments be non stationary? Still, to the best of our knowledge, the claim of unstirred components is used as a figure of merit of stirrer efficiency only over one position [5], without checking how it fares from a global point of view. In other words, the current approach could lead to the conclusion that a stirrer is effectively generating a proper random sequence at one position and not at another.

Moreover, the ACF cannot be properly estimated from the N_s samples provided by the stirring technique at one position in most real-life configurations. Its statistical uncertainty can be assessed by means of the laws recalled in [10]: as the correlation gets weaker, the uncertainty can be overwhelmingly higher than the actual value to estimate. As we will show that even weak residual sample correlations play a central role in GoF tests, the relatively high uncertainty affecting its estimate when working on a single position would not allow to draw any useful information.

We would rather suggest to regard an RC as a multivariate random generator. Now the field generated over N_p different positions for a given stirrer configuration will be modeled as a random vector, treated as a single random realization $\boldsymbol{X}_i = (x_{i,1}, ..., x_{i,N_p})$, with $\boldsymbol{X}_i \in \mathbb{C}^{N_p \times 1}$. The sample average will now be referred to as

$$\hat{\boldsymbol{\mu}} = \langle \boldsymbol{X}_i \rangle_{N_p} \quad , \quad (3)$$

with the arithmetic average applied to each of the N_s scalar components $x_{i,j}$. The sample covariance matrix $\hat{\boldsymbol{C}}$ will be given by

$$\hat{\boldsymbol{C}} = \frac{N_p}{N_p - 1} \left\langle (\boldsymbol{X}_i - \hat{\boldsymbol{\mu}})(\boldsymbol{X}_i - \hat{\boldsymbol{\mu}})^{\mathrm{H}} \right\rangle_{N_p} \quad (4)$$

and the sample correlation between two stirrer configurations

$$\hat{\rho}_{i,j} = \frac{C_{i,j}}{\sqrt{C_{i,i} C_{j,j}}} \quad (5)$$

now evaluated over N_p positions.

This type of approach allows studying the behavior of an RC in a global way, rather than the local one currently used when looking for unstirred components. Having access to a broader view is fundamental in the estimate of the residual correlation affecting the stirring technique, and thus the accuracy of the estimate of eventual unstirred components. As a matter of fact, the stirrer efficiency should be evaluated as its ability to generate uncorrelated samples over an entire range of positions within the chamber, more specifically taken within its test volume. The same goes for the eventual existence of unstirred components.

Another valid representation would have consisted in switching the role of the stirrer realizations and the spatial ones. Although entirely valid, this description would lead to a correlation matrix assessing the correlation between the samples taken at different positions, i.e., the spatial correlation of the field within an RC. As opposed to the modelling of the correlation between stirrer-generated samples, the spatial correlation quantity features a good agreement between experimental data and theoretical models [11]. Hence, it is expected to take relatively low values as the distance between the probe position go beyond the wavelength and changes sign as the distance increases. As a result, its average value in space can be expected to be much lower than the correlation between the samples generated by the stirring technique. This assumption is supported by experimental results presented in Section V. As a consequence, the alternative description of mixing together spatial- and stirring-obtained data should not be regarded as a valid approach when correlation data are being examined, since their correlations are remarkably different, as will be shown in Section V.

III. On the detection of an offset

Following this discussion, each realization $\boldsymbol{X}_i = (x_{i,1}, ..., x_{i,N_p})$ will be assumed to have elements following the same family of probability law, but not necessarily with the same statistical moments. No assumption will be invoked on the moments of $\boldsymbol{X}$ in this work: the only assumption will be that the $x_{i,j}$ be normally distributed. Such requirement should actually not be regarded as a constraint, as it is part of the definition of the null hypothesis we are going to detail in the next section.

Let us recall, for the time being, that the assumption of an overmoded cavity implies that the field samples should be indeed normally distributed, with a zero average value, independently from the use of an efficient stirrer, or for that of a stirrer at all [12], a well-known fact in acoustics. It is thus natural to consider that the field samples should be normally distributed, even though a LoS component is present: this latter will only affect the arithmetic average

$$\bar{X}_i = \langle \boldsymbol{X}_i \rangle_{N_s} \tag{6}$$

of the N_s samples generated by the stirrer at the i-th position.

The typical approach to estimate the intensity (and presence) of an eventual LoS contribution is to compute $\bar{X}_i$ at a given position. It is thus interesting to study its behavior in the framework presented in the previous section. The offset $\bar{X}_i$ can be regarded as a random variable, whose ensemble average can be shown to be

$$\mu_{\bar{X}} = \mathsf{E}\left[\bar{X}_i\right] = \langle \boldsymbol{\mu} \rangle_{N_s} \quad , \tag{7}$$

with $\mathsf{E}\left[y\right]$ the ensemble average of the random variable y and $\boldsymbol{\mu}$ the ensemble average of the multivariate random variable $\boldsymbol{X}$. The ensemble average should not be confused with the arithmetic averages typically used as estimates of statistical moments. The arithmetic average in (7) is taken over the N_s scalar values constituting the vector $\boldsymbol{\mu}$.

The variance of the offset estimator can be shown to be

$$\sigma^2_{\bar{X}} = \mathsf{E}\left[|\bar{X}_i|^2\right] = \bar{\sigma}^2 \left(\frac{1}{N_s} + \bar{\rho}\right) \quad , \tag{8}$$

where

$$\begin{aligned} \bar{\sigma}^2 &= \langle C_{i,i} \rangle_{N_s} && \text{(9a)} \\ \bar{\rho} &= \mathrm{Re}\,\langle \rho_{i,j} \rangle_{N_s(N_s-1)} \quad , && \text{(9b)} \end{aligned}$$

having taken the arithmetic average in (9b) over all of the off-diagonal elements of the covariance matrix $\boldsymbol{C}$. The quantities $\bar{\sigma}^2$ and $\bar{\rho}$ are the average variance and correlation factor of the field measured within the RC over the N_s stirrer realizations and N_p positions.

Following the assumption of a multivariate normal law, the offset estimator follows a normal law, too, with mean and variance given in (7) and (8), respectively. It is fundamental to recall that the assumption of normality is not linked to the use of a stirring technique. The normality of the field samples is asymptotically expected in any overmoded cavity, even in a static configuration [12]. This fact will play a fundamental role in the next section, when defining an alternative hypothesis test. In particular, the assumption of a normally distributed $\bar{X}$ does not require to invoke the central limit theorem (CLT).

Getting back to (8), the presence of a correlation term ensures that the CLT and therefore the law of large numbers cannot be invoked, as the N_s scalar random variables cannot be assumed to be independent. Whence, even if $N_s \to \infty$, the uncertainty over the estimate $\bar{X}$ of the offset can never go to zero. Indeed

$$\lim_{N_s \to \infty} \sigma^2_{\bar{X}} = \bar{\rho}\bar{\sigma}^2 \quad , \tag{10}$$

leading to a lower bound in the accuracy of the estimator. This lower bound is far from negligible: for what is usually regarded as a negligible correlation, e.g., $\bar{\rho} = 0.1$, the reduction of the original standard deviation in the samples is translated into a mere reduction of about a factor 3 in the standard deviation of the offset estimator. This is far from being a satisfying accuracy gain obtained by collecting a large number of samples.

This is where the current approach to detecting unstirred components fail. As a matter of fact, the GoF tests usually applied are based on the assumption of iid samples. Such assumption has profound implications: indeed, it implies that for an increasing number of samples N_s, the accuracy of the data-set is increasingly fine. In other words, GoF tests tend to ignore imprecise data, especially those with low probability, as N_s keeps sufficiently low. But as N_s increases, these statistics tends to become more demanding, leading to rejections of the null hypothesis as soon as small imperfections take the data set apart from the reference one. In the presence of a finite correlation, the statistics of the GoF tests are incapable of acknowledging the fact that further data do not provide new complementary information for the rejection of the null hypothesis.

The problem is that the estimation of the residual correlation is typically based on one single observation point, thus affected by a large statistical uncertainty. Moreover, it is often assumed that a sufficiently low correlation factor implies that the data are “almost” independent [10]. Equation (8) clearly shows that even the smallest correlation (the real one, not the estimate) rules out the use of the CLT. In other words, the actual value of the offset $\bar{X}$ cannot be accessed. This residual uncertainty is actually in contradiction with the assumptions at the base of currently used GoF tests. As N_s increases, they do expect the accuracy of the estimate to improve. Hence a misconception and misuse of these tools. In a practical way, the CLT and hence most of the GoF tests can be used only as long as

$$N_s \ll \frac{1}{\bar{\rho}} \quad , \tag{11}$$

so that the usual reduction in the statistical uncertainty predicted by the CLT holds. For larger populations, the results from GoF tests are going to be too conservative, as the actual amount of information provided by the data is less than might be expected for uncorrelated samples.

It could be argued that this limitation can be bypassed by extracting a subset of independent samples from the original samples. In fact, as demonstrated by the very definition of the effective sample size, the amount of information in the two sets, and in particular the uncertainty it involves in the estimation of the ensemble average is the same [10]. Hence, this limitation would subsist. Moreover, the concept of independent samples is controversial, as it is not related to a threshold value in the residual correlation, nor to a clearly defined procedure that allows to extract in a precisely defined manner these samples. The actual approach suggested in international standards [7] and recent works [10] is just to decimate the original population, which does not ensure the independence of the the remaining samples with certainty.

IV. An alternative hypothesis test

The presence of an offset can now be restated in an alternative manner. Let us consider the null hypothesis of a perfect RC, presenting all of the features required by asymptotic models [9], in particular a zero-average. We are interested in assessing what is the probability of observing a sample $\boldsymbol{X}_i$ at a given position presenting an offset $\bar{X}_i$, although its actual ensemble average is equal to zero.

Under these hypothesis, a standardized random variable ζ can be introduced

$$\zeta = \frac{\bar{X}}{\sigma_{\bar{X}}} = \frac{\bar{X}}{\bar{\sigma}} \left(\frac{1}{N_s} + \bar{\rho} \right)^{-1} \quad , \tag{12}$$

following (8). As the null hypothesis consider that the chamber is ideal, the field samples will follow a normal law, and thus ζ too. It is therefore possible to state that the presence of an unstirred component should be considered as statistically significant to a probability α only if the value taken by ζ at one position (single realization) is larger than the quantile q_α of a standardized normal distribution. In any other case, the fact that $\bar{X}_i \neq 0$ should not be regarded as due to unstirred components.

It is noteworthy that this definition accounts for the existence of a residual correlation between the samples. Indeed, as the average correlation increases, ζ is scaled down, so that it becomes less likely to decide for the presence of an unstirred component only on the basis of a non-negligible offset. The decision on the actual presence of an unstirred component should be based on a stronger foundation, i.e., by comparing the estimated offset with the uncertainty affecting its estimation. To this effect, a z-test could be applied, since ζ is a standardized normal variable, thus rejecting the null hypothesis if

$$|\zeta| > \frac{q_{1-\alpha/2}}{\sqrt{N_p}} \quad , \tag{13}$$

where $q_{1-\alpha/2}$ is a quantile for the standard normal distribution. It could be argued that the z-test requires independent samples. This is true, but since the samples ζ_i are now taken over N_p different positions in space their residual correlation can be expected to be far lower than that of a stirring technique, if these positions are sufficiently far apart. This assumption will be experimentally shown to hold in the next section.

The strength of this approach lies in the fact that the validity of the proposed model can be checked from an experimental point of view. Indeed, the quantities appearing in (12) can be estimated from measurements: if the experimentally-derived values of ζ do appear to follow a standardized normal distribution, than our modeling approach should be regarded as sound. It is fundamental to notice that the role of $\bar{\rho}$ has a dramatic impact on the behavior of ζ, even for small values. This will be show in the next section to be the reason why we regard the proposed test as physically and statistically correct.

V. Experimental results

We carried out measurements in Supelec's RC, with dimensions $3.08 \times 1.84 \times 2.44$ m, with a lower usable frequency of about 550 MHz as defined in [7]. We considered the frequency range between 1 and 3 GHz, in order to ensure a negligible probability of incurring into a configuration where the RC would not behave as an overmoded cavity. Moreover, at these frequencies our mechanical stirrer is electrically large and aperiodic, so that it can ensure low correlation between samples if sufficiently large angular steps are considered: the 50 angular samples used provided an estimated correlation

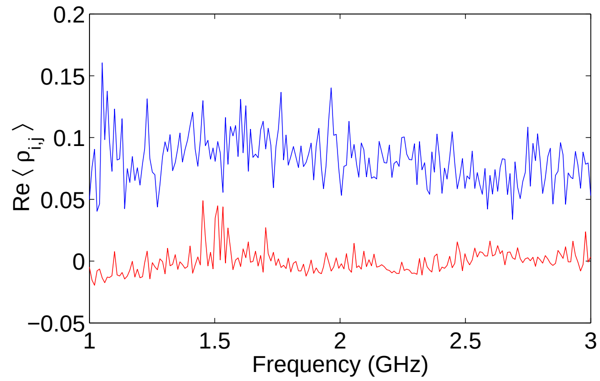


Fig. 1. Residual correlation $\bar{\rho}$ as defined in (9), computed from experimental data when considering the stirrer (blue) or the positions (red) as random realizations of multivariate samples.

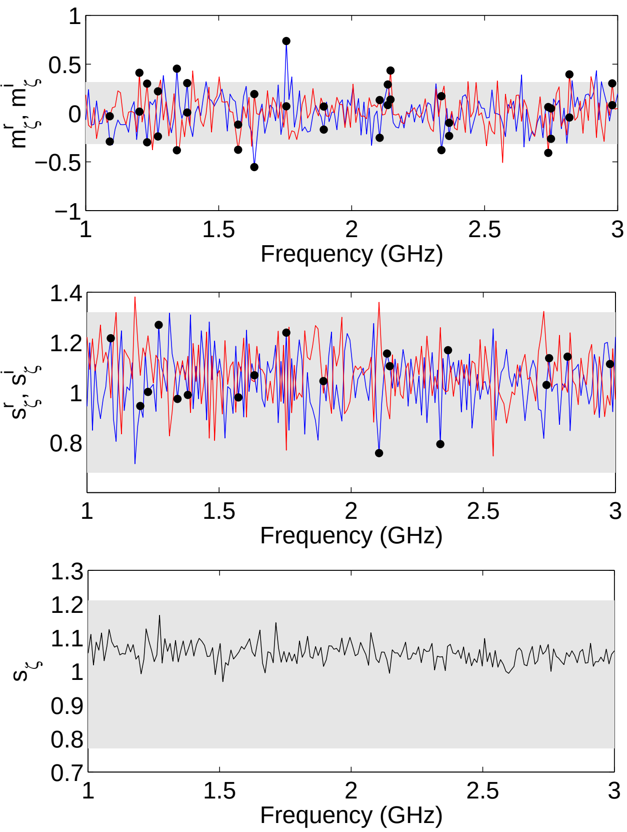


Fig. 2. Empirical moments of ζ estimated from the N_s multivariate samples: (a) real (blue lines) and imaginary (red lines) parts of the sample average (m_ζ^r, m_ζ^i); (b) standard deviations for the real and imaginary parts of ζ (s_ζ^r, s_ζ^i); (c) standard deviation of ζ (s_ζ). Shaded areas represent the expected confidence margin at 95 % probability for the null hypothesis, for a sample population of N_p realizations.

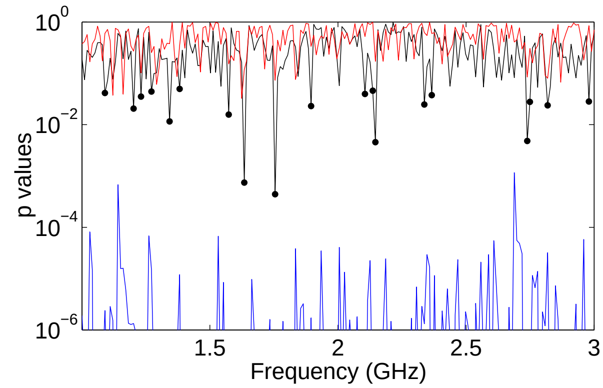
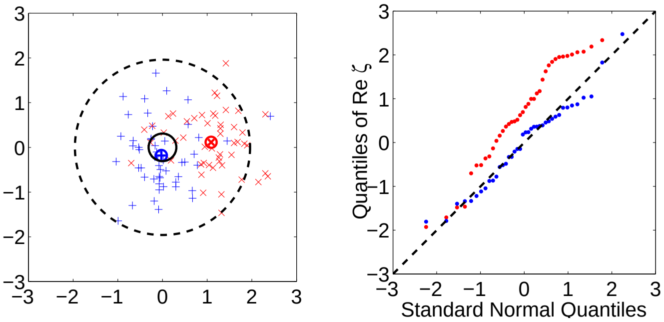


Fig. 3. Probabilities of rejection of the null hypothesis, as obtained by means of three different approaches: (a) acceptance of a KS test applied to univariate data over all of the N_p positions (minimum of the N_p p-values), in blue; (b) null hypothesis of $\mathrm{Re}\zeta$ and $\mathrm{Im}\zeta$ as standard normal variables, in black; (c) null hypothesis of $|\zeta|$ behaving as a χ_2 variable. The black dots stand for the frequencies over which the test (b) is rejected with a significance level $\alpha = 0.05$ for either the real or the imaginary parts.

below 0.3. A log-periodic antenna was used to excite the field within the chamber, oriented towards a corner in such a way as to have its main lobe scattered towards the stirrer. We can thus regard as unlikely that unstirred components in the sense given in [3] be present. A horizontal component of the electric field (normal to the sample plane) thus generated was measured by means of an electro-optic field probe (Enprobe EFS-105), capable of preserving the phase of the field while providing a negligible scattering cross-section: it was thus possible to move it freely within the RC without any risk of altering the actual field distribution. Forty positions were considered, distributed over a 5×8 (vertical and horizontal, with a step of 37.5 cm and 25 cm, respectively) Cartesian grid dividing into two halves the chamber along its longer dimension.

As proposed in Section II, we treated the spatial samples for a given stirrer position as one single realization of a multivariate random variable. The empirical covariance and correlation matrices were computed for this case and for the complementary one, where the role of the stirrer and spatial samples are switched. This yielded the $\bar{\rho}$ results presented in Fig. 1: it is indeed proven that the average correlation in much lower among the spatial samples than between the stirrer ones, as anticipated in Section III. As a result the assumptions on the modeling of the ζ random variable hold, so that its definition is statistically sound. The empirical moments of ζ where than computed, and are shown in Fig. 2: as required by our assumptions, ζ presents a unitary variance. This result should not be underestimated, as neglecting the residual correlation $\bar{\rho}$ would have led to a twofold increase of the variance of ζ, which would have led to a higher rate of rejection of the null hypothesis. The fundamental role played by $\bar{\rho}$ even for weak values is thus also confirmed.

The consistency of the statistical properties of the field distribution were then tested against the zero-mean assumption expected from their asymptotic modeling. Three approaches were applied: 1) the minimum p-value was considered for Kilmogorov-Smirnov (KS) GoF tests based on the N_s-sample population at each of the N_p positions in space; 2) the real and imaginary parts of ζ were tested for normality through KS tests over N_p samples; 3) $|\zeta|$ was tested through KS against a χ_2 distribution. The results presented in Fig. 3 show that indeed, if the presence of "unstirred" components is considered as unacceptable at one position, than our RC does not allow to generate an unbiased field distribution over a region of space. This overreaction of the KS test comes from the fact that the N_s samples measured over one single position are hardly independent, and that since the condition in (11) is not satisfied, the effect of the correlation is not negligible. Another important issue is the fact that testing against χ_2 (and in general any function of the modulus of the field components) does not allow assessing whether the real and imaginary parts have identical features. Indeed, the test based on the $|\zeta|$ is much less strict than that applied to the real and imaginary parts of ζ. Taking a significance level $\alpha = 0.05$, leads to the identification of a number of frequencies where the average value of the field generated within the RC can no more be



(a) (b)

Fig. 4. The ζ_i samples measured at 1.744 GHz (blue) and 1.754 GHz (red): (a) scatter plot of the ζ_i samples, with their average values marked by circles. The confidence limits for $\alpha = 0.05$ are shown for the case of a single local measurement (solid line) and for the global 40-sample population (dashed line); (b) quantile-quantile plots based on a comparison with a standard normal random variable.

explained through statistical uncertainty alone.

An example is given in Fig. 4a, where the scatter plots of the $N_s = 40$ population of ζ_i are shown for two close frequencies, 1.744 GHz and 1.754 GHz, where the ζ-test null hypothesis is respectively accepted and rejected. The KS test rejected the null hypothesis of normal distributions respectively in 27 and 37 configurations for these two frequencies: it is hence not possible to assess whether the RC presents major differences at these two frequencies. Indeed, the scatter plots clearly show the huge uncertainty in the estimate of the LoS component, as the confidence margins associated to the null-hypothesis for a single local measurement do not lead to a clear decision scenario. At 1.754 GHz, only 5 samples lie outside the confidence margin for the local test. Conversely, the global test provides a much stricter decision criterium, as the arithmetic averages of the two populations are wide apart from the smaller confidence region. Such global approach indicates without doubts that at 1.754 GHz, the RC is not behaving as expected, with the ζ_i clearly biased away from the origin. The proposed approach indeed allows assessing in what scenario the RC is found: at 1.744 GHz our RC presented a zero-mean ζ, well explained by statistical uncertainty alone, as soon as residual correlation is taken into account.

A last result of interest is presented in Fig. 4b, where the quantile-quantile plots of the real part of the ζ_i are considered against a standard normal variable (null hypothesis) at the two same frequencies as before. This figure shows that at 1.754 GHz the rejection of the ζ test is not only motivated by the existence of an offset, since the plot is not close to a line, but strongly distorted. In other words, removing its average value does not lead to the test being accepted. This points to the fact that apart in atypical configurations as those considered in [3], the field distribution is not merely modified by a LoS term, but the field distribution is likely superposed to a stationary wave that is weakly interacting with the stirrer. This would better explain the non constant bias and the statistics distortion at the same time, since the field does not present stationary moments in space. This could also likely be linked to a loss of field uniformity.

VI. Conclusions

This paper has introduced an alternative approach in the detection of "unstirred" components. Rather than applying the usual local approach, a more general one was employed, based on the modeling of an RC as a multivariate random generator. Studying the statistical properties of the moments of this model, it was shown that weak correlation values should never be neglected, particularly when testing experimental data against theoretical reference ones.

The introduction of the test distribution ζ capable of taking into account this residual correlation has led to a sounder test, whose physical validity has been tested against experimental data, thus pointing to the intrinsical importance of taking into account the residual correlation. Experimental results also proved that the reason of rejection of the null hypothesis cannot be entirely put on the presence of a bias, since the appearance of a profoundly different statistical behavior of the RC points to deeper reasons not well understood.

Future work will be needed in order to establish a proper testing approach on a global scale, rather than on a local one, in particular in order to set a relationship between the number of positions needed and the accuracy of the test.

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